Non-linear robust control for inverted-pendulum 2D walking

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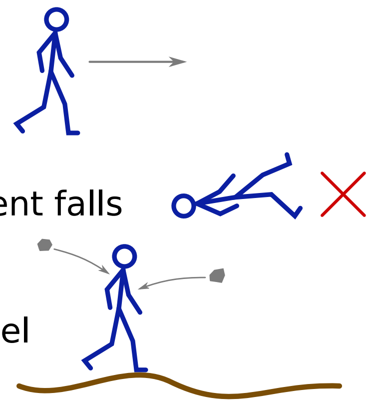
Robust Walking:

- **objective:** regulate speed
- **constraint:** completely prevent falls
- **given:** disturbance model



Dynamics:

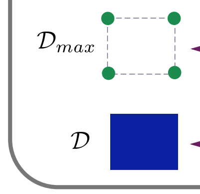
$$\omega^- = \sqrt{(\omega_k)^2 + \frac{2g}{\ell}(1-\cos\phi)}$$ swing-down

$$\omega^+ = (\omega^-)\cos 2\phi + \frac{2p}{ml}\cos\phi\sin\phi$$ push-off, heel-strike

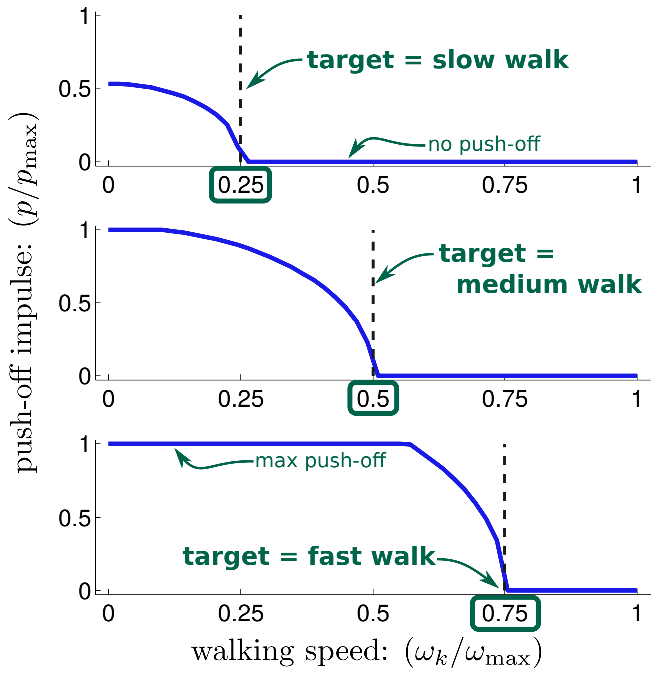
$$\omega_{k+1} = \sqrt{(\omega^+)^2 - \frac{2g}{\ell}(1-\cos\phi)}$$ swing-up

Disturbances:

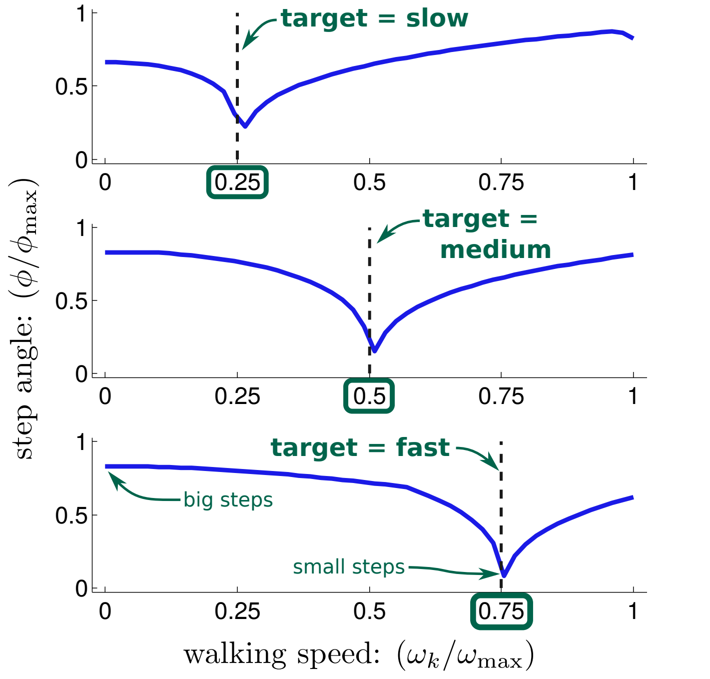
$\ell := \ell + \delta_\ell,$	leg length	$\lvert\delta_\ell\rvert \le \Delta_\ell$
$p := p + \delta_p,$	push-off	$\lvert\delta_p\rvert \le \Delta_p$
$\phi := \phi + \delta_\phi,$	step length	$\lvert\delta_\phi\rvert \le \Delta_\phi$
$\omega_k := \omega_k + \delta_\omega,$	hip speed	$\lvert\delta_\omega\rvert \le \Delta_\omega$



Push-off controller

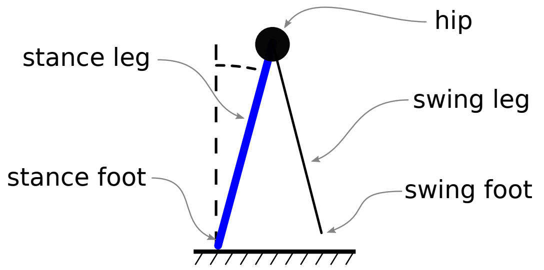


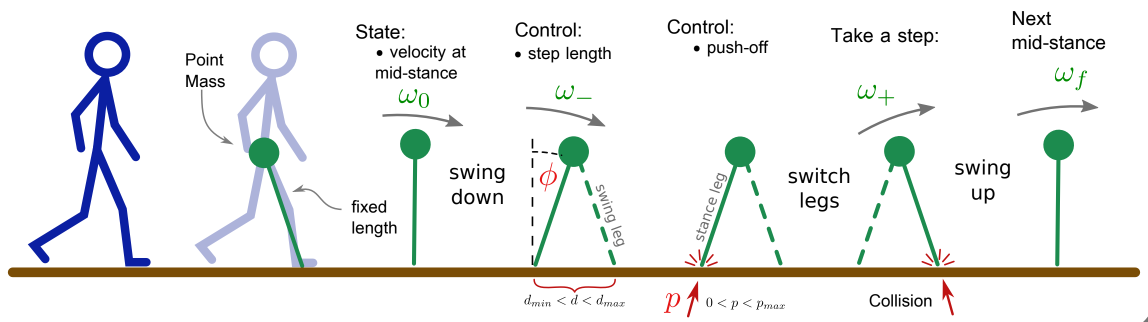
step length controller



Model:

- point-mass hip
- inextensible leg
- impulsive push-off





Controller:

$$\{p, \phi\} = K(\hat{\omega}_k)$$

Lyapunov function:

$$V(\omega_k) = (\omega^* - \omega_k)^2$$

$$V(\omega_{k+1}) < V(\omega_k) \quad \forall \omega_k \in \Omega_I$$

Objective function:

$$f(p,\phi) = \sum_{\boldsymbol{\delta}_i \in \mathcal{D}_{\max}} (\omega^* - \omega_i)^2$$

Robust stability:

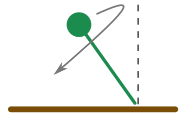
$$V(\omega_{k+1}) < V(\omega_k) \qquad \forall \omega \in \Omega_I - \Omega_g, \quad \forall \boldsymbol{\delta} \in \mathcal{D}$$

$$V(\omega_G) \le V(\omega_B) \qquad \forall \omega_G \in \Omega_G, \quad \forall \omega_B \in \delta\Omega_G, \quad \forall \boldsymbol{\delta} \in \mathcal{D}$$

Subject to

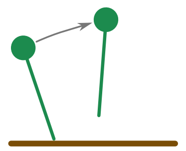
Constraints:

no falling back



$$\omega^+ > \sqrt{\frac{2g}{\ell}(1-\cos\phi)}$$

no flight



$$\omega^- < \sqrt{\frac{g}{\ell}\cos\phi}$$ **before push-off**

$$\omega^+ < \sqrt{\frac{g}{\ell}\cos\phi}$$ **after push-off**

$$0 < (2m\ell)(\omega^-)\cos\phi\sin\phi - p\left(\cos^2\phi - \sin^2\phi\right)2\ell$$ **during push-off**

Table: Parameters

Symbol	Name	Value
m	mass	9.91 kg
g	gravity	9.81 m/s^2
ℓ	leg length	0.96 m
$p_{\max}$	max push-off impulse	12.2 kgm/s
$\phi_{\max}$	max stance angle	$30° = 0.52$ rad
$\omega_{\max}$	max mid-stance speed	2.56 rad$/s$
Δ_ℓ	leg length error bound	$\pm$ 0.05 ℓ
Δ_p	push-off error bound	$\pm$ 0.05 $p_{\max}$
Δ_ϕ	step length error bound	$\pm$ 0.05 $\phi_{\max}$
Δ_ω	mid-stance speed error bound	$\pm$ 0.05 $\omega_{\max}$

Controller Verification